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OWNR problem with variable replenishment and inventory holding costs and quantity discounts

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Abstract

We consider an inventory/distribution system consisting of one warehouse and N retailers (OWNR). Warehouse replenishes its orders from external supplier and supplies all orders of retailers. The objective of this paper is to maximise supply chain benefits of the whole system. To achieve this objective SENYIGIT and AKKAN's heuristic algorithm is used. We assume that demand is constant and deterministic; shortages are not allowed and lead times are negligible. Total system cost consists of order-quantity-dependent replenishment and inventory holding cost parameters, and quantity discounts are applicable. The main idea of new heuristic is to compare replenishment and inventory holding costs while manipulating the order quantity by order intervals of locations. While balance condition is searched by new version of SENYIGIT and AKKAN's heuristic algorithm, quantity discounts and replenishment costs will reduce costs of whole systems. Capital limit constraint is also included to the model of objective function. The algorithm is developed for quantity discounts and 100 datasets randomly generated for the system that contains five retailers and the solutions were obtained by developed heuristic.

Keywords: Supply chain management, two echelon inventory systems, integer-ratio policies, quantity discounts, heuristic method, variable cost parameters.

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1. Introduction

Over the last decades, companies know that the prices are decreasing and costs are increasing worldwide. That is because decreasing cost is mandatory to sustain the companies' life cycle. Companies tend to supply chain management to decrease the costs. This management skill worked for some years and many researchers worked on supply chain management to decrease the costs. So many methods have been developed. However, all companies in the supply chain stages started to consider the benefits of their own companies. This individuality did not make the whole system work properly, and made companies consider the whole system benefits and sustainability because of increasing competition.

For proper and efficient movement of products from suppliers to the ultimate customers, customer-oriented supply chain management is necessary. Customer orientation makes companies to deliver the products on time, to correct address and with correct quantity. Also, total system costs must be the minimum. To achieve this objective, integrated supply chain must be established at all stages of supply chain. Therefore, research on supply chain stock policies is converted into multi-stage–multi-locations instead of one stage–one locations. First step of these research studies is one warehouse and *N* retailers (OWNR) distribution systems to decrease the total cost of the system. To solve the first step, it is assumed that the demand is constant and deterministic, lead times are negligible, shortages are not allowed, fixed replenishment and inventory holding costs, quantity discounts are not available, infinite capitals, one to multiple two-stage distribution supply chain, single product, etc.

2. Literature review

Roundy (1985) showed, in some situations that optimal nested policies can have very low effectiveness. For that reason, he dropped the assumption of nestedness and analysed a more general class of policies referred to as integer-ratio policies. In the integer-ratio policies, the replenishment interval at the warehouse is t_0 and the replenishment interval at retailer t_j, $j = 1,2,3, \ldots, N$ must satisfy that either t_0/t_jor t_j/t_0 a positive integer. Notice that following this policy, a retailer can order less frequently than the warehouse. In addition, the warehouse does not necessarily order the same quantity each time an order is placed. Therefore, integer-ratio policies are always stationary at the retailers but may not be at the warehouse. In particular, Roundy (1985) focused on the integer-ratio policies that are also powers-of-two. In such policies, orders are placed at equal intervals of time that are powers-of-two multiples of a base planning period. Roundy (1985) showed that the cost of an optimal powers-of-two policy is at most 2% above the optimal cost.

Abdul Jalbar (2010) considered integer-ratio policies between order intervals of warehouse and retailers and generated a heuristic algorithm on this policy. There is an integer-ratio between warehouse and retailers' order intervals. In that paper, heuristic algorithm works on $(r_j) = 0,1,2, \ldots, N$ ratio between replenishment and inventory holding costs. (r_j) Parameter gives the decision for changing frequency of warehouse or retailers order interval. If $1, 2 \geq (r_j) \geq 0,4$ number interval can be obtained for any retailer, appropriate solution can be found. If $r_j = 1$ can be found, that is an optimal solution. The method was run for randomly generated data and it is observed that for many examples $r_j$could not be in the desired number interval and the algorithm could not finish the loop.

Senyigit and Akkan (2012) used integer policies for t_j order intervals and developed a new heuristic searches t_j individually and together for all stock holding locations. They could obtain optimal solutions after heuristic search solutions. And they proved optimality with the rule of Abdul Jalbar (2010) $r_j = 1$ equality. This new heuristic method could find the minimum system cost compared with the other methods.

3. Problem statement

In this study, we consider to include some other cost parameters to the objective function and capital limit constraint to the constraints and reform the heuristic method to solve the new type of OWNR distribution problem.

In this problem, retailers supply the ultimate customers' demand and warehouse supply retailers' demand occurred by the ultimate customers. An external supplier that is not included to the problem supplies the warehouse demand occurred by retailers. All retailers have a constant and deterministic demand. Shortages are not allowed. Lead times are negligible. Variant replenishment and inventory holding costs are considered with deterministic and experimental values. Quantity discounts are available. Problem is established for single type product and distribution has two stages, which is one to multi-distribution. Retailer and warehouse capitals are finite.

Problem Parameters	
j	**Stock location index**
N	**Number of stock locations**
f_j	$\forall\, j = 0, 1, 2, \ldots .\, N$ **Total replenishment number of stock location j**
t_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Order intervals of stock location j**
d_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Demand per unit time for stock location j**
μ_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Variable coefficient of quantity discount depend on order quantity**
k_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Replenishment cost of stock location j per order**
θ_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Variable coefficient of replenishment costs depend on order quantity**
h_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Inventory holding cost of stock location j per unit and per unit time**
ω_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Variable coefficient of inventory holding costs depend on order quantity**
Q_j	$\forall\, j = 0, 1, 2, \ldots, N$ **The quantity of products replenished at stock location j**
SP_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Selling Price of Product**
PP_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Purchasing Price of Product**
C_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Total Cost incurred in stock location j**
PC_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Unit Cost incurred in stock location j**
B_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Total Profit incurred in stock location j**
S_j	$\forall\, j = 0, 1, 2, \ldots, N$ **Capital limit of stock location j**
B_T	**Total System Profit**
C_T	**Total System Cost**

The assumption of constant and deterministic demand means that the inventory levels of the retailers decrease linearly. According to this assumption, we can say that holding inventory level is equal to average demand of retailer during order interval(t_j). However, warehouse inventory level is not as the retailers'. Because the retailers have different cost parameters and demand quantity, so this causes different order frequency for each retailer. These differences can increase and decrease stock level at warehouse at different times. So, the inventory level graphic of warehouse does not appear as retailers' (linear). It is considered that warehouse inventory level is dependent on a relation between order intervals of each retailer and warehouse. This relation has a constraint that makes the system solve by heuristic methods. If warehouse replenishment interval is greater than replenishment interval of retailer *j*, retailer *j* replenishes more frequently than warehouse. So, warehouse holds required inventory for retailer *j* for time horizon. If warehouse replenishment interval is greater than replenishment interval of retailer *k*, warehouse

does not need to hold inventory for retailer k. This relation makes objective function dependent on solutions. If then we include in objective function warehouse inventory holding cost due to retailer j. Otherwise, we do not. Therefore, before solutions have been found the objective function cannot be written.

Objective function in Senyigit and Akkan (2012) is determined as cost minimisation. Quantity discounts are integrated to new problem definition and it is expected that unit cost will decrease. But even unit cost decreases, total cost can be increased because of greater amount of order quantities. So that can generate mistakes to find better solutions and not to make mistakes it is better to convert objective function to profit maximisation instead of cost minimisation. For profit calculation, we need to know purchasing and selling prices and these parameters are also included to problem definitions. And also, capital limits are defined for stock locations to interrupt the maximisation.

Objective function conversion also allows to manipulate the selling prices according to seasonal demand policies. Additionally, assumptions and constraints are added in the objective functions and mathematical model of the problem.

Retailer total replenishment cost: $\forall\ j = 1, 2, \ldots, N$ retailers total replenishment cost at base planning period is equal to $k_j \times f_j$ and by $f_j \times t_j = 1$ equation k_j/t_j. θ_j is variable coefficient of replenishment cost and for retailer j

$\forall\ j = 1,2, \ldots, N$ total replenishment cost is $k_j\theta_j/t_j$.

Retailer inventory holding costs: It is assumed that the demands of retailers are constant and deterministic. The inventory held between the two orders of intervals is equal to $d_j/2$. So, inventory holding cost at retailer *j* $\forall\ j = 1, 2, \ldots, N$ is equal to $h_j d_j t_j/2$. ω_j is variable coefficient of inventory holding cost and for retailer *j* total inventory holding cost is $h_j\omega_j d_j t_j/2$.

Warehouse replenishment cost: Total replenishment cost at base planning period is $k_0 \times f_0$. With $f_0 \times t_0 = 1$, equation total replenishment cost is converted into k_0/t_0. θ_0 is variable coefficient of replenishment cost and for warehouse total replenishment cost is $k_j\theta_j/t_j$.

Warehouse inventory holding cost: The relation between warehouse and retailers differ the formulation of warehouse holding cost from economic order quantity formulation. As we discussed before, warehouse holds inventory just for $t_0 > t_j$ retailers. This means warehouse holds the sum of the inventory for retailer *j* whose order interval is smaller than warehouse order interval ($t_0 > t_j$). Warehouse inventory holding cost function can be written $h_0 \sum_{j:\, t_0 > t_{j.}} \frac{(t_0 - t_j) d_j}{2}$. ω_0 is variable coefficient of inventory holding cost and for warehouse total inventory holding cost is $h_0 \omega_0 \sum_{j:\, t_0 > t_{j.}} \frac{(t_0 - t_j) d_j}{2}$.

It is not known which retailer's order interval is longer than warehouse order interval and which one is shorter. Objective function changes itself after solutions due to warehouse holding inventory cost formulation. Depending on whether the condition between t_0 and is provided, warehouse holding inventory module for retailer *j* is included to the objective function or not. This makes objective function cannot be written exactly before solutions have been found. So, the solution of the function is dependent on the solutions. This causes heuristic methods.

4. Solution algorithm

Senyigit and Akkan (2012) heuristic method finds integer values of optimal solutions, and then by using these values, it is possible to find the optimal solutions.

Objective function is as follows.

$Max\ B_T$

$$B_T = \sum_{j=0}^{N} B_j = \sum_{j=0}^{N} \{d_j SP_j - C_j\}$$

$$C_j = \frac{k_j}{t_j}\theta_j + \frac{h_j d_j t_j}{2}\omega_j + PP_j d_j \mu_j$$

$$C_0 = \frac{k_0}{t_0}\theta_0 + \left\{ t_0 > t_j \,\middle|\, h_0 \omega_0 \sum_{t_0 > t_j ; j=1}^{N} \frac{(t_0 - t_j) d_j}{2} \right\} + PP_0 d_0 \mu_0$$

$$B_T = \sum_{j=0}^{N} B_j = \underbrace{\sum_{j=1}^{N} \left\{ d_j SP_j - \left[\frac{k_j}{t_j}\theta_j + \frac{h_j d_j t_j}{2}\omega_j + PP_j d_j \mu_j \right] \right\}}_{Retailers'\ Total\ Benefit}$$

$$+ \underbrace{\sum_{j=1}^{N} PP_j d_j \mu_j - \left[\frac{k_0}{t_0}\theta_0 + \left\{ t_0 > t_j \,\middle|\, h_0 \omega_0 \sum_{t_0 > t_j ; j=1}^{N} \frac{(t_0 - t_j) d_j}{2} \right\} + PP_0 d_0 \mu_0 \right]}_{Warehouse's\ Total\ Benefit}$$

$$d_0 = \sum_{j=1}^{N} d_j$$

$$S_j \geq C_j \qquad \theta_j, \omega_j, \mu_j, t_j > 0$$

$$j \in \{0, 1, 2, \ldots, N\}$$

We assume that warehouse purchasing price (PP_0) is 3.90 U.S. dollars without discount and retailer purchasing price (PP_j) is 5.00 U.S. dollars without discount. Table 2 shows the discrete values of quantity discounts calculated with ($\boldsymbol{\mu_j}$) variable coefficient of quantity discounts. Discrete values of ($\boldsymbol{\mu_j}$) are randomly generated. While manipulating ($\boldsymbol{t_j}$) values in Senyigit and Akkan (2012) heuristic search ($\boldsymbol{Q_j}$) order quantity value is replaced and ($\boldsymbol{\mu_j}$) gets the matched value between lower and upper bound range.

Table 1. Warehouse and retailer variable coefficient of quantity discounts

Warehouse quantity discounts range				Retailer quantity discounts range			
Order quantity lower bound	Order quantity upper bound	Unit price	(μ_0)	Order quantity lower bound	Order quantity upper bound	Unit price	(μ_j)
0	1,000	$3.90	1.00	0	100	$5.00	1.00
1,001	1,400	$3.67	0.94	101	200	$4.85	0.97
1,401	1,900	$3.35	0.86	201	300	$4.60	0.92
1,901	2,500	$3.04	0.78	301	400	$4.45	0.89
2,501	-	$2.89	0.74	401	-	$4.30	0.86

Table 3 shows the discrete values of ($\boldsymbol{\omega_j}$) variable coefficient of inventory holding costs. These values are randomly generated. While manipulating ($\boldsymbol{t_j}$) values in Senyigit and Akkan (2012) heuristic search ($\boldsymbol{Q_j}$) order quantity value is replaced and ($\boldsymbol{\omega_j}$) gets the matched value between lower and upper bound range.

Table 2. Warehouse and retailer variable coefficient of inventory holding cost

Warehouse inventory holding quantity range			Retailer inventory holding quantity range		
Holding quantity Lower bound	Holding quantity Upper bound	(ω_j)	Order quantity Lower bound	Order quantity Upper bound	(ω_j)
0	1,000	1	0	100	1
1,001	1,400	0.96	101	200	0.95
1,401	1,900	0.93	201	300	0.89
1,901	2,500	0.89	301	400	0.79
2,501	-	0.84	401	-	0.68

We assume that replenishment can be reached by Lorries and trucks. A lorry has 500 units capacity and a truck has 800 units. The given cost of replenishment in examples is for Lorries. And it is fixed that truck load is 50% more expensive than a lorry load. The variable coefficients of replenishment cost (θ_j) are given in Table 4.

Table 3. Variable coefficient of replenishment cost of truck and lorry

Variable coefficients of replenishment cost for lorry						Variable coefficients of replenishment cost for truck					
Lower bound	Upper bound	(θ_j)	Lower bound	Upper bound	(θ_j)	Lower bound	Upper bound	(θ_j)	Lower bound	Upper bound	(θ_j)
0	100	0.25	2,501	2,600	5.25	0	100	0.3	2,501	2,600	5.05
101	200	0.49	2,601	2,700	5.49	101	200	0.55	2,601	2,700	5.27
201	300	0.7	2,701	2,800	5.7	201	300	0.77	2,701	2,800	5.47
301	400	0.88	2,801	2,900	5.88	301	400	0.97	2,801	2,900	5.65
401	500	1	2,901	3,000	6	401	500	1.15	2,901	3,000	5.8
501	600	1.25	3,001	3,100	6.25	501	600	1.3	3,001	3,100	5.92
601	700	1.49	3,101	3,200	6.49	601	700	1.42	3,101	3,200	6
701	800	1.7	3,201	3,300	6.7	701	800	1.5	3,201	3,300	6.3
801	900	1.88	3,301	3,400	6.88	801	900	1.8	3,301	3,400	6.55
901	1,000	2	3,401	3,500	7	901	1000	2.05	3,401	3,500	6.77
1,001	1,100	2.25	3,501	3,600	7.25	1001	1100	2.27	3,501	3,600	6.97
1,101	1,200	2.49	3,601	3,700	7.49	1101	1200	2.47	3,601	3,700	7.15
1,201	1,300	2.7	3,701	3,800	7.7	1201	1300	2.65	3,701	3,800	7.3
1,301	1,400	2.88	3,801	3,900	7.88	1301	1400	2.8	3,801	3,900	7.42
1,401	1,500	3	3,901	4,000	8	1401	1500	2.92	3,901	4,000	7.5
1,501	1,600	3.25	4,001	4,100	8.25	1501	1600	3	4,001	4,100	7.8
1,601	1,700	3.49	4,101	4,200	8.49	1601	1700	3.3	4,101	4,200	8.05
1,701	1,800	3.7	4,201	4,300	8.7	1701	1800	3.55	4,201	4,300	8.27
1,801	1,900	3.88	4,301	4,400	8.88	1801	1900	3.77	4,301	4,400	8.47

1,901	2,000	4	4,401	4,500	9	1901	2000	3.97	4,401	4,500	8.65
2,001	2,100	4.25	4,501	4,600	9.25	200	2100	4.15	4,501	4,600	8.8
2,101	2,200	4.49	4,601	4,700	9.49	2101	2200	4.3	4,601	4,700	8.92
2,201	2,300	4.7	4,701	4,800	9.7	2201	2300	4.42	4,701	4,800	9
2,301	2,400	4.88	4,801	4,900	9.88	2301	2400	4.5	4,801	4,900	9.3
2,401	2,500	5	4,901	5,000	10	2401	2500	4.8	4,901	5,000	9.55

Solutions will be obtained by New Version of Senyigit and Akkan (2012) heuristic method. Heuristic method has to be rearranged for new problem and cost parameters. For sensitivity analysis, all parameters that effect solution will be randomly generated.

5. Conclusion and further studies

Our purpose in this study is to introduce a new solution for OWNR problem. We researched variable parameters for real life situations. After solutions have been found it is considered to include multi-warehouses in the problem objective function.

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