Apps for Solving Engineering Problems Using Numerical Techniques

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Abstract

Some mathematical software offers capabilities for designing personalized graphical interfaces. Users that interact with these tools only have to load data to obtain results, avoiding code frequently hard to understand. One of the best advantages of these tailor-made windows is that students can focus on the issue being studied, instead of the code/language used. The discussion of the code may be the next step. Taking advantage of these possibilities, the research Group GIE (Grupo Ingeniería & Educación, in Spanish) of the Universidad Tecnológica Nacional from Argentina has been developing, since 2008, a collection of tailor-made windows related to the different issues of Numerical Analysis. The use of these tools in class is guided so as conceptual discussions arise, to make students appreciate the importance of the application of numerical methods.
This paper shows some of the new tools developed with Mathematica for solving different engineering problems using numerical methods.

Keywords: Virtual Laboratories, Apps, Numerical Analysis

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1. Introduction

The evaluation criteria set by the National Commission for University Evaluation and Accreditation (CONEAU, stands for Comision Nacional de Evaluacion y Acreditacion Universitaria, in Spanish) in Argentina emphasize the importance assigned to problem solving in engineering education for careers’ accreditation. More precisely, this committee highlights the competence of graduates to solve problems as an indicator of the quality of the education offered by an institution. Thus, the idea that the engineer must know how to solve problems is a basic postulate that supports and guides the curriculum decisions in the career and actions in the classroom (Jover, 2003).

In particular, in Numerical Analysis courses of engineering careers at Facultad Regional San Nicolás, Universidad Tecnologica Nacional, in Argentina, the different issues will be introduced by posing engineering problems. This new methodological approach, based on the theory of Mathematics in the context of Sciences, has contributed to the significant teaching of certain mathematical concepts. Among them, the design of a course of differential equations in the context of electrical circuits (Camarena Gallardo, 1987), the Fourier series in the context of the process of mass transfer (Muro & Camarena Gallardo, 2002) and the Laplace transform in the context of Engineering (Suarez & Camarena Gallardo, 2000) can be mentioned.

The aim of this paper is to present some visual tools designed with MATHEMATICA for solving specific problems related to different engineering specialties that are modelled by nonlinear equations. The authors believe that introducing each unit with this kind of situations is suitable, as prior students’ knowledge is activated, besides the need of learning numerical methods is generated in students.

2. Mathematics in Science context

Mathematics in Science Context is a theory that considers the link that should exist between mathematics and the sciences that require it. It is based on the facts that knowledge arises integrated and Mathematics is a supportive tool and a formative discipline that has a specific function in university courses (Camarena Gallardo, 2009).

Diverse factors involved in teaching and learning processes give rise to the five phases of the theory of Mathematics in Science Context: curricular, didactic, epistemological, educational training and cognitive. In mathematics learning environments at university, these five phases are present and interact with different effects over each other.

Mathematics in context helps students to build their own knowledge in a solid and lasting way because this methodology enables the development of mental skills through the process of solving problems related to the chosen career.

Within the didactic phase, Mathematics in Science context proposes a professional skill development methodology in order to solve contextual situations. In this way, it is possible to develop abilities to transfer knowledge. This didactic methodology has three stages:

- The introduction of Mathematics in Context didactic strategies in the learning environment.
- The implementation of extracurricular courses where activities take place as a way to develop thinking skills, metacognitive skills, and apply heuristics to solve problems, as well as activities to block negative beliefs.
- The instrumentation of an integral and interdisciplinary workshop in the last semesters of the career, where real industry events are solved.
The first stage includes a teaching strategy known as Mathematics in Context, which deals with making students live a mathematics linked to their interest, without artificial applications, that is to say, a mathematical contextualized in the areas of knowledge of their future profession.

Mathematics in Context proposal may be implemented in several ways, one of them is via problem resolution. The resolution of contextual problems might be used to accomplish different objectives: diagnosis, motivational, knowledge formation, evaluation, among others. Mathematics in context enabled students to build long lasting structured and unfractionated knowledge, with firm and lasting ties.

3. Teaching Numerical Analysis in Engineering context

Engineers frequently need to solve problems that do not have an exact solution. It is interesting to use these situations to introduce the topics to be developed in Numerical Analysis courses. In this way, students not only acquire skills and abilities to numerically solve these problems but also to analyze why it is necessary to learn certain concepts in their professional training.

To carry out this methodological proposal based on solving engineering problems, various visual tools were designed.

Some mathematical software offers capabilities for designing personalized graphical interfaces. Users that interact with these tools only have to load data to obtain results, avoiding many lines of code, frequently hard to understand. One of the best advantages of these tailor-made windows is that students can focus on the issue being studied, instead of the code/language used. The discussion of the code may be the next step.

Taking advantage of these possibilities, the research Group GIE (Grupo Ingeniería & Educacion, in Spanish) of the Universidad Tecnologica Nacional from Argentina has been developing, since 2008, a collection of tailor-made windows related to the different issues of Numerical Analysis included in different courses at the Facultad Regional San Nicolás, from the Universidad Tecnologica Nacional.

At the beginning, Maple was the software chosen for the development of personalized windows, then new windows were developed with SciLab, a free software. As Wolfram offered the possibility of creating CDF files that did not need Mathematica to run, the group started to develop some tools as CDF files. Only the CDF player is needed, available for free on Internet.

In the following sections, the problems that have been selected for introducing nonlinear equations together with the tools designed are presented. It is noteworthy that all the tools were prepared for Spanish-speaking students, so the images captured and shown in this paper are not in English.

3.1. Numerical methods involved when solving nonlinear equations

Diverse methods with different characteristics are available for solving nonlinear equations of one variable, \( f(x) = 0 \), among them bisection and Newton stand out for different reasons: unconditional convergence the former, rate of convergence the latter. To apply bisection method, necessary conditions for Bolzano theorem are needed: continuity of the function and an initial interval containing the root to be determined, with different sign of the function at its ends. To apply Newton’s method, first and second continuous derivative of the function are required.

Different criteria exist to stop these iterative methods. When convergence is a fact, weak convergence may be a way to do it, by comparing successive results. In this case, as the aim is to find an approximation of the value \( x^* \) that makes \( f(x^*) = 0 \), and for the methods studied continuity is a requirement of the function, the criteria for stopping the method is that the absolute value of the function evaluated in the approximation should be less than a tolerance established.

3.2. A tool for evaluating the internal rate of return
The evaluation of an investment project is carried out to decide its acceptance or rejection, or to compare two different projects (Sapag Chain & Sapag Chain, 1991).

In order to perform an objective assessment, economic indicators are used. One of these indicators is the internal rate of return (IRR) that can be calculated from the net present value (NPV) of the project. A rate of return for which the net present value is zero is an internal rate of return. The NPV of the project is:

\[ NVP = \sum_{k=0}^{n} \frac{C_{f_k}}{(1+i)^k} \]  \hspace{1cm} (1)

where \( n \) is the number of years of the project, \( i \) is the rate of return of the project and \( C_{f_k} \) is the cash flow corresponding to the \( k \) year. Therefore, the equation that allows finding the IRR is:

\[ 0 = \sum_{k=0}^{n} \frac{C_{f_k}}{(1+IRR)^k} \]  \hspace{1cm} (2)

As equation (2) is nonlinear, numerical methods must be used to solve it. The tool shown in Figure 1 calculates the IRR of an investment project applying the bisection method. Despite this method is not the most efficient, it has the advantage of unconditional convergence.

In order to get a result in this app, users need to provide data inherent to the problem: the cash flows projected in each period. Then, data inherent to the method: the tolerance, and an initial interval containing the root to be calculated. So as to facilitate this latter task, at the bottom of the window, a graph of the function associated with the equation to be solved is shown.

Figure 1. A tool for evaluating the internal rate of return
3.3. A tool for analyzing four-bar linkage mechanisms

A four-bar linkage mechanism can be modeled by a nonlinear function. Considering one bar lying along the x-axis, this mechanism is the one shown in Figure 2. The angle \( \alpha \) is the input to this mechanism, and the angle \( \phi \) is the output. A relationship between \( \alpha \) and \( \phi \) can be obtained by writing the vector loop equation, where \( r_1 \) lies along the x-axis (Hoffman, 1992):

\[
 r_2 + r_3 + r_4 - r_1 = 0 
\]

Figure 2. A four-bar linkage mechanism

Equation (3) can be written as two scalar equations, corresponding to the x and y components of the \( r \) vectors. Therefore,

\[
 r_2 \cos(\theta_2) + r_3 \cos(\theta_3) + r_4 \cos(\theta_4) - r_1 = 0 
\]  
\[
 r_2 \sin(\theta_2) + r_3 \sin(\theta_3) + r_4 \sin(\theta_4) = 0 
\]

Combining equations (4.a) and (4.b), letting \( \theta_2 = \phi \) and \( \theta_4 = \alpha + \pi \), and simplifying, yields Freudenstein’s equation (Hoffman, 1992):

\[
 R_1 \cos(\alpha) - R_2 \cos(\phi) + R_3 \cos(\alpha - \phi) = 0 
\]

where

\[
 R_1 = \frac{r_1}{r_2}, \quad R_2 = \frac{r_1}{r_4} \quad \text{and} \quad R_3 = \frac{r_1^2 + r_2^2 - r_3^2 + r_4^2}{2r_2r_4} 
\]

Considering the four bar linkage specified by \( r_1 = 10, r_2 = 6, r_3 = 8 \) and \( r_4 = 4 \), equation (5) becomes

\[
 \frac{5}{3} \cos(\alpha) - \frac{5}{2} \cos(\phi) + \frac{11}{6} - \cos(\alpha - \phi) = 0 
\]

As it can be seen in Equation (7), \( \alpha \) and \( \phi \) are related in this equation in a nonlinear way. So to obtain \( \phi \) for a given value of \( \alpha \), a nonlinear equation must be solved.

In Figure 3, a tool for obtaining an approximate solution of \( \phi \) is shown. Data of the problem must be selected, from the drop-down lists at the left side of the window: values of the length of the bars and the input angle.
It is noteworthy that the pre-loaded lengths of the bars satisfy the Grashof Law: the sum of the lengths of the shortest and longest bars (4 and 1 respectively), cannot be greater than the sum of the other two bars (2 and 3). On the right side of the window, the approximate values of the successive iterations are shown.

The tool presented in Figure 3 may be used as a motivating activity for analyzing the sufficient conditions of the Fourier Law, which ensures convergence of Newton’s Method. First, students can select as initial approximation a value observed on the graphic of the function associated to the equation to be solved. As there exists a minimum of the function relatively near to the searched root, the question about which conditions should be satisfied by the initial point selected to guarantee the convergence of the method arises.

### 3.4. A tool for studying real gases

An ideal gas obeys the law:

\[ P \ V = n \ R \ T \]  \hspace{1cm} (8)

where \( V \) is the gas volume, \( P \) is the pressure, \( n \) is the amount of gas moles, \( R \) is the gas constant and \( T \) is the absolute temperature.

If the gases were ideal, product \( P \ V \) should be constant at any pressure but all gases deviate from this behavior in most conditions. In general, the curve \( P \ V \) as a function of \( P \) for a real gas passes
through a minimum. In lighter gases such as hydrogen and helium, and in all gases at temperatures far above the boiling point, this minimum is not observed. In all gases there is a temperature, known as Boyle temperature, where the minimum of the curve \( f(P) = PV \) disappears. However, the minimum becomes very visible near the condensing temperature (Gray & Haight, 1976).

Different equations to predict the behavior of real gases have been proposed. The best known is the Van der Waals equation. This is an equation of state for real gases that takes into account the forces of attraction and repulsion between molecules. Thus, the equation of an ideal gas, \( PV = nRT \), is transformed into:

\[
(P + \frac{a}{V^2})(V - b) = RT
\]

where \( V \) is the volume of a mol. The values \( a \) and \( b \) can be determined from the critical constants of gases.

Figure 4 presents the designed tool to obtain \( V \), if \( P \) and \( T \) are known for a specific gas using the Van der Waals equation. Data of the problem to be solved -gas, pressure and temperature- should be selected from the drop-down lists at the top of the window. The gases used in the application are nitrogen, oxygen, carbon dioxide, ammonia, sulfur dioxide and ethylene (Glasstone, 1979).

Newton's method is used to make the calculations. A tolerance and the maximum iterations allowed must be selected. The initial value to start applying the method is obtained using the corrected equation of the ideal gases:

\[
\frac{V_0}{a} = b + \frac{RT}{P}
\]

This tool is adequate to raise in class an analysis of the stop criteria of an iterative process.

![Figure 4. A tool for studying real gases](image)

4. Conclusion

In numerical analysis courses, students often ask themselves . . . Why do we have to study numerical analysis? The apps shown here give an answer to this question. The authors of this paper
think that posing engineering problems like the ones mentioned before, allows students to feel that learning numerical analysis is a necessity, not an obligation, thus pointing to the psychological significance of learning. Therefore, it is important to introduce each issue studied by posing a situation where the students discover that the math they have learnt before is not sufficient.

Solving problems with tools that students can interact with, allowing them to obtain results according to different parameter values, makes them protagonist of their learning process, instead of being a mere spectator. Clearly, professors should intervene appropriately, guiding students to achieve the objectives set at the beginning of each unit. For 2017 academic year, new tools will be designed for different topics.

References